the photoemission measurements are characteristic of Cu-rich Cu-Ni alloys because the samples used in this study were not homogenized. Nevertheless, the comments of Seib and Spicer show with some confidence that the virtual-bound-state model is more likely to be the right one. It would be very desirable to repeat the photoemission measurements, this time using well-homogenized samples which are subsequently very quickly quenched from high temperature to room temperature.

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Supplementary Results on "Low-Temperature Resistivity of Dilute Magnetic Impurities in the Presence of Internal Fields"

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Previous calculations on the effect of random internal fields on the Kondo resistivity are extended to cover a wider range of variation of internal field parameters and exchange constants. An error in the previous calculated results is also corrected.

In a previous work[1] we have examined the low-temperature resistivity associated with scattering of electrons by magnetic impurities dispersed in a nonmagnetic solvent. The calculation was done in an effective field approximation using the second Born approximation to obtain the relaxation times. Because of an error[2] in one of the expressions, we recalculated the values of the resistivity and have taken this opportunity to extend our calculations to a wider range of values of the s-d exchange constant J. In the previous calculation,[1] a particular choice of the width of the truncated Lorentzian field distribution Δ was used, where Δ is proportional to the impurity concentration. Whereas in the previous work Δ/c was taken to be a constant determined from specific-heat data, in the present paper we regard Δ/c as a parameter, and give the resistivity for several values of Δ/c. This will allow for effects of differing geometrical and degeneracy factors in the internal fields entering the resistivity and the specific-heat calculations. The distribution of internal fields is still assumed to be independent of the temperature. This is not strictly correct,[3] but is reasonable at low temperatures. The general features of the resistivity calculations will be recalled: As the temperature is lowered from the T^5 region the resistivity exhibits a minimum at $T_{\min}$ and then rises approximately logarithmically in temperature to a maximum at $T_{\max}$, finally falling approximately linearly at still lower temperatures.[4] For higher concentrations of magnetic impurities the extrema in the resistivity are replaced by inflection points.

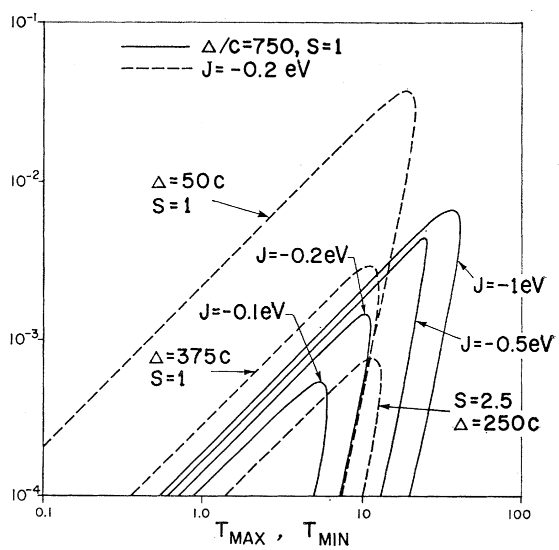

FIG. 1. Values of $T_{\max}$ and $T_{\min}$ as a function of the impurity concentration, where $T_{\max}$ is the temperature of the resistivity maximum and $T_{\min}$ that of the minimum. The lower temperature is associated with the maximum. The solid lines all have the same internal field parameter Δ/c and S, and indicate the variation with J, whereas the dashed lines all have the same value of J.

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[1] R. J. Harrison and M. W. Klein, Phys. Rev. **154**, 540 (1967).
[2] R. J. Harrison and M. W. Klein, Phys. Rev. **167**, 878(E) (1968).
[3] M. W. Klein, Phys. Rev. **173**, 552 (1968).
[4] It is assumed throughout that the temperature is such that perturbation theory is valid. See Footnote 20, Ref. 1. Also see R. More and H. Suhl, Phys. Rev. Letters **20**, 500 (1968).

The present calculations exhibit quite completely the dependence of the values of $T_{\max}$ and $T_{\min}$ upon the concentration and the other parameters in the problem, namely the values of S, J, and Δ/c. We find that the qualitative features of the resistivity are not changed by the error in the formulae in Ref. 1. Figure 1 shows the relationship between $T_{\max}$ and $T_{\min}$ and the impurity concentration. The linear portions at low concentrations on this logarithmic plot exhibit on the one hand the $c^{1/5}$ dependence of $T_{\min}$ originally described by Kondo[5] which is only slightly modified by internal field effects. On the other hand, the internal effects are all-important for $T_{\max}$ and yield the linear dependence upon c already discussed.[1] It is also implicit in our previous discussion that the value of $T_{\max}$ will depend linearly upon Δ/c. The values of $T_{\max}$ and $T_{\min}$ depart from these two

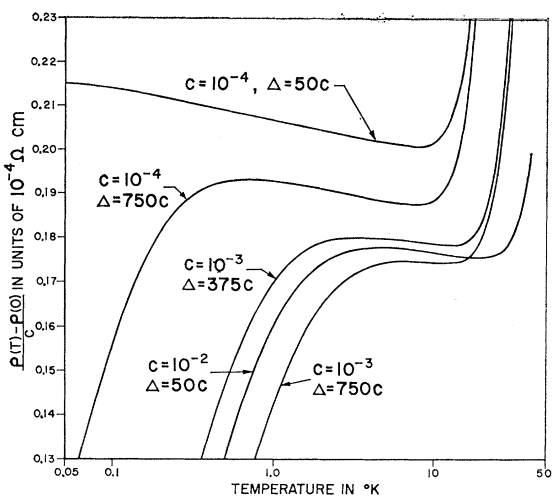

FIG. 2. The change in the resistivity $\rho(T)$ from its nominal value at $T=0$ divided by the concentration c as a function of the temperature for several values of c and of parameter Δ.

straight lines near the region of their projected intersection. It may in fact be noted that the shapes of the curved portions of the loci of the temperature of the extrema in Fig. 1 are all very closely the same, indicating that there is an approximate universal relationship providing that both temperature and concentration are normalized to values depending only upon J, S, and Δ/c.

Figure 2 presents curves of resistivity versus temperature for an assorted variety of parameters, exhibiting the features discussed in the above paragraph. The low-temperature resistivity well below the maximum is still dominated by a term linear in T. Figure 3 shows the effect of varying S for a given concentration, while

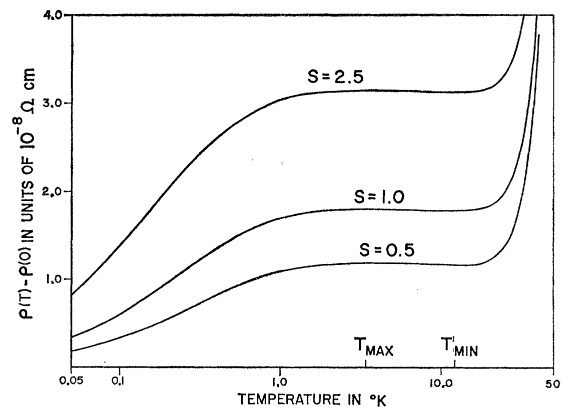

FIG. 3. The change in resistivity $\rho(T)$ from its nominal value at $T=0$ value as a function of the temperature for several values of S, where S is the impurity spin. The values of J and Δ were adjusted such that the temperature of the maximum and minimum coincide for the three values of S indicated.

adjusting the values of J and the internal field parameter constant Δ/c to yield the same values of $T_{\max}$ and $T_{\min}$ for the differing S values. With minor adjustments determined from Fig. 1, the value of J was chosen to fit the value of $T_{\min}$, and then the value of Δ/c chosen to fit the value of $T_{\max}$, taking into account that the change in $T_{\min}$ for a change in S followed a $\frac{1}{5}$ power of $S(S+1)$ while $T_{\max}$ seemed to go, less precisely, as the $\frac{2}{5}$ power. The value of S determines the height of the approximate plateau between the extrema with respect to the value of the resistivity at much lower temperatures. On the other hand, the incremental resistivity in the region between the extrema is closely the same for all S, even on a greatly expanded scale. Thus there is no important difference that could be used to discriminate between values of S if the experimental data is confined to this region.

In conclusion, we have presented calculations of the low-temperature resistivity which may permit comparison with the experimental resistivity and aid determination of the parameters. In further discussion, we may remark that in considering finite concentration effects, there are two types: the dynamical effect and the statistical effect. The dynamical effect of a single pair of impurities has been recently treated by Béal-Monod[6] in the second Born approximation. On the other hand, we have treated here the statistical aspect of the problem using an effective field approximation. Béal-Monod had given a pre-Kondo qualitative discussion of the statistical aspect of this problem,[7] indicating that a low-temperature maximum can arise from internal field effects, and we regret not having made reference to this in our previous work.[1]

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[7] M. T. Béal, J. Phys. Chem. Solids **25**, 543 (1964).